Technical Comments

Comment on "Flow of Dilute Polymer Solutions in Rough Pipes"

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Nomenclature

A = log law constant
B = log law constant for Newtonian fluid and smooth wall
c = polymer concentration
$F(k^+)$ = $-\Delta B$ for uniform sand-grain roughness and Newtonian fluid (Poreh's model)
$F_{as}(k^+)$ = asymptotic form of $F(k^+)$
G_{as} = $\Delta B_m + F_{as}$
k = sand-grain roughness size; $k^+ = kv_*/\nu$
$p(k/\delta)$ = relative roughness size function in Poreh's model
R = pipe radius
x = dummy variable
$u(z)$ = mean longitudinal velocity; $u^+ = u/v_*$
v_* = friction speed, $(\tau_w/\rho)^{1/2}$
v_{*crit} = friction-reduction threshold value of v_*
z = distance normal to wall; $z^+ = zv_*/\nu$
$\alpha(c)$ = parameter in Meyer's model
δ = nominal viscous sublayer thickness
ΔB = shift in log law constant due to polymer and/or wall roughness
ΔB_m = ΔB for smooth wall case with polymer (Meyer's model)
ν = kinematic viscosity (approximated by the solvent value)
ρ = fluid density
$\phi(x)$ = operator on x, such that $\phi(x) = x, x \geq 1$; $\phi(x) = 1, x < 1$
τ_w = wall shear stress

Introduction

FOR friction-reducing, dilute polymer solutions in turbulent pipe flow, the shifts in the log law-of-the-wall constants A and B, due to the effects of polymer and wall roughness, are important to drag-reduction predictions for external flows. For the smooth-wall, Newtonian-fluid case, the mean velocity profile log law is

$$u^+ = A \log z^+ + B$$

With polymer and/or wall roughness, it is often assumed that A remains fixed, so that

$$u^+ = A \log z^+ + B + \Delta B$$

Poreh[1] has proposed a ΔB model for the combined effects of polymer and wall roughness, starting with Meyer's model[2] for the smooth wall case:

$$\Delta B = \Delta B_m = \alpha(c) \log[\phi(v_*/v_{*crit})]$$

where $\alpha(c)$ and v_{*crit} are dependent on the polymer type, and ϕ is an operator such that

$$\phi(x) = x, x \geq 1 \qquad \phi(x) = 1, x < 1$$

Poreh's model is a blending of ΔB_m with a model for the case of uniform, sand-grain roughness and Newtonian fluid, obtained by curve fitting to Nikuradse's pipe flow data. Poreh also considered nonuniform roughness, but we consider only uniform roughness here.

In using Poreh's model, we have found that it predicts a peculiarly abrupt and severe onset of roughness effect for low values of kv_{*crit}/ν. Since this anomaly seems to be unsupported by any experimental data, we have analyzed Poreh's model, found that the anomaly arises from an insignificant detail of the roughness-only model, and then considered two other possible models which give a more reasonable, smooth onset of roughness effect. One of the two models is at least compatible with some very limited experimental data[3] and seems to merit further consideration.

Analysis

In order to enlarge Meyer's model to include the effect of uniform sand-grain roughness on ΔB, Poreh first fitted a model for the Newtonian-fluid case to Nikuradse's data, obtaining (for $A = 5.75$ and $B = 5.5$)

$$\Delta B = -F(k^+), \quad F(k^+) = 0, \quad k^+ < 3.35$$
$$F(k^+) = 0.26(k^+ - 3.35) - 0.0026(k^+ - 3.35)^2 \qquad 3.35 < k^+ < 20 \tag{1}$$
$$F(k^+) = A \log[k^+ - 2.0 - 17.4/(k^+)^{1/2}] - 3.0 \qquad 20 < k^+$$

In order to introduce his concept of relative roughness size, Poreh rewrites Eq. (1) as

$$\Delta B = -F_{as}(k^+)p(k/\delta) = -F_{as}(k^+)p(k^+/\delta^+)$$
$$F_{as} = A \log[\phi(k^+/k_0{}^+)], A \log k_0{}^+ = 3.0, k_0{}^+ = 3.325$$
$$p(k/\delta) = 0, \text{ for } k/\delta < 3.35/11.6 = 0.29$$
$$p(k/\delta) = \{0.26[11.6(k/\delta) - 3.35] - 0.0026[11.6(k/\delta) - 3.35]^2\}/F_{as}, \text{ for } 0.29 < k/\delta < 20/11.6 = 1.72 \tag{2}$$
$$p(k/\delta) = \{A \log[11.6(k/\delta) - 2.0 - 17.4/(11.6\, k/\delta)^{1/2}] - 3.0\}/F_{as}, \text{ for } 1.72 < k/\delta$$

The nominal viscous sublayer thickness δ is taken by Poreh as the height of the intersection of the viscous sublayer and log laws, while ignoring any effect of roughness, so that

$$\delta^+ = A \log\delta^+ + B + \Delta B_m \tag{3}$$

In the roughness-only case or whenever $v_* \leq v_{*crit}$, $\delta^+ = 11.6$ for $A = 5.75$ and $B = 5.5$, so that Eq. (2) reduces to Eq. (1).

A plot of $p(k/\delta)$ vs $\log(k/\delta)$ is given in Fig. 1. The abrupt behavior at $k/\delta = 0.29$ reflects the fact that the zero of $F(k^+)$ at $k^+ = 3.35$ is very close to the zero of F_{as} at $k^+ = k_0{}^+ = 3.325$. Thus, while both F and F_{as} vary smoothly, their ratio $p(k/\delta)$ varies abruptly as $F \rightarrow 0$.

Poreh's model for combined polymer and roughness effect is

$$\Delta B = \Delta B_m - G_{as}p(k/\delta), \quad G_{as} = \Delta B_m + F_{as} \tag{4}$$

Figure 2 gives a plot of ΔB vs $\log k^+$ according to Eq. (4) for the case of $\alpha = 17.2$; the figure should be essentially the same as Fig. 4 of Ref. 1 except for the additional low values of kv_{*crit}/ν. Those low values bring out clearly the peculiarly abrupt onset of roughness effect mentioned earlier, but it is also present (or should be) for $kv_{*crit}/\nu = 3$ and 2.

The cause of the anomaly is clearly the $p(k/\delta)$ behavior for k/δ at and near 0.29, as inspection of Figs. 1 and 2 suggests.

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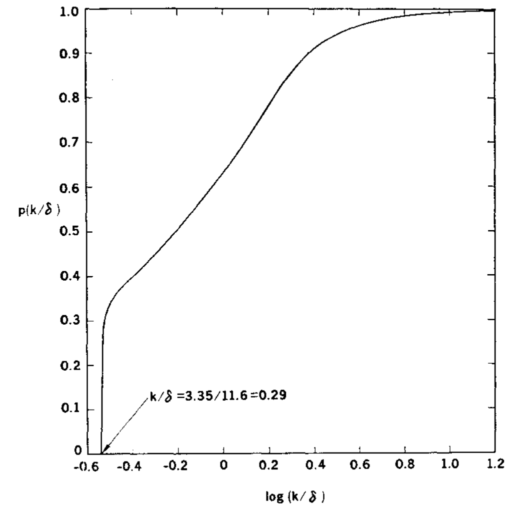

Fig. 1 Plot of $p(k/\delta)$ vs $\log(k/\delta)$.

That behavior is more and more reflected in the ΔB curves as kv_{*crit}/ν decreases from 3.35.

The reasons why the abrupt onset of roughness effect depends on kv_{*crit}/ν in the way it does can be seen as follows. Since

$$k/\delta = k^+/\delta^+ = (v_*/v_{*crit})(kv_{*crit}/\nu)/\delta^+ = 0.29(v_*/v_{*crit})[(kv_{*crit}/\nu)/3.35]11.6/\delta^+ \quad (5)$$

then for $v_* = v_{*crit}$, $\delta^+ = 11.6$ according to Eq. (3) and

$$k/\delta = 0.29[(kv_{*crit}/\nu)/3.35]$$

This shows that the onset of roughness effect precedes the attainment of the critical wall shear stress if $kv_{*crit}/\nu > 3.35$, and follows it if $kv_{*crit}/\nu < 3.35$.

When roughness effect precedes, $\Delta B_m = 0$ and $\delta^+ = 11.6$ up to $k^+ = kv_{*crit}/\nu > 3.35$, so that ΔB from Eq. (4) reduces to ΔB from Eq. (1). In other words, the abrupt behavior of $p(k/\delta)$ is suppressed by $F_{as}(k^+)$, and we recover the smooth behavior of $F(k^+)$ up to $k^+ = kv_{*crit}/\nu$. On the other hand, as kv_{*crit}/ν decreases below 3.35, the value of k^+ at the roughness-effect onset increases, as substitution of Eq. (3) into Eq. (5) for $k/\delta = 0.29$ shows. (The dashed curve in Fig. 2 is the resultant onset locus.) Thus, since

$$G_{as} = \alpha \log\{\phi[k^+/(kv_{*crit}/\nu)]\} + A \log[\phi(k^+/k_0^+)]$$

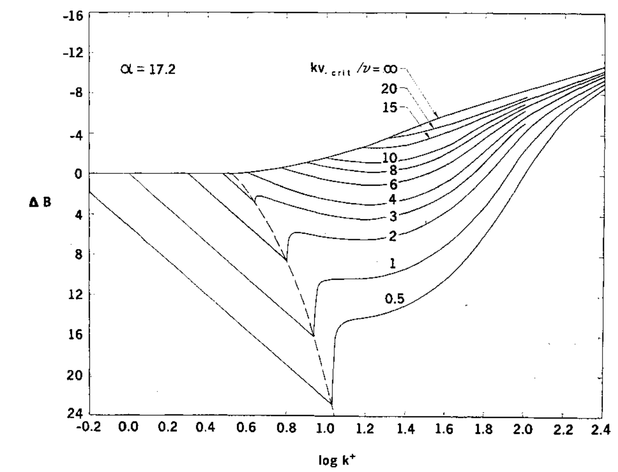

Fig. 2 ΔB vs $\log k^+$ according to Poreh's model for α = 17.2 and various values of kv_{*crit}/ν.

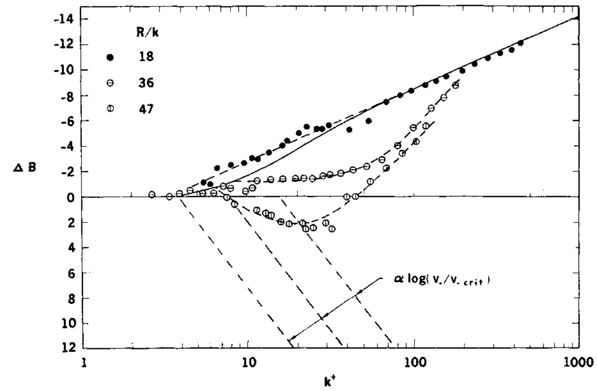

Fig. 3 Experimental data of Spangler[3] for ΔB vs k^+ for three pipe flow cases with uniform roughness for α = 17.2, and v_{*crit} = 0.0968 fps.

the onset value of G_{as} increases with the onset value of k^+ and magnifies the abrupt behavior of $p(k/\delta)$.

Thus, the anomalous behavior of Poreh's model arises from an otherwise insignificant detail of the fitting to Nikuradse's data. On this ground alone, the model can be questioned. Furthermore, if Nikuradse's data were faired slightly differently, $p(k/\delta)$, and hence ΔB, would be discontinuous at the onset of roughness effect. The relative roughness size idea of roughness-effect onset at a critical value of k/δ can be questioned also. At least if Newtonian fluid behavior in the viscous sublayer is assumed, with the top of the roughness element initially below the nominal sublayer thickness, since $3.35 < 11.6$, it is unreasonable to expect the onset value of k^+ to depend on δ^+ as δ^+ increases.

A Different Approach

In order to avoid the extreme sensitivity of Poreh's model to a detail of the roughness-only model, let us consider models for combined polymer and roughness effect of the form

$$\Delta B = -F(k^+) + \Delta B_m'(k^+,\alpha,v_*/v_{*crit}) \quad (6)$$

where $\Delta B_m'$ reduces to ΔB_m for $k^+ = 0$. In particular, consider

$$\Delta B_m' = \alpha(c)r_1 \log[\phi(v_*/v_{*crit})] \quad (7)$$

and

$$\Delta B_m' = \alpha(c) \log[\phi(r_2 v_*/v_{*crit})] \quad (8)$$

where r_1 and r_2 are to be determined experimentally. Hopefully, r_1 or r_2 will be found to be independent of polymer type and concentration and only dependent on k^+. However, these questions cannot be investigated here, and we only consider whether r_1 or r_2 is compatible with the available data.

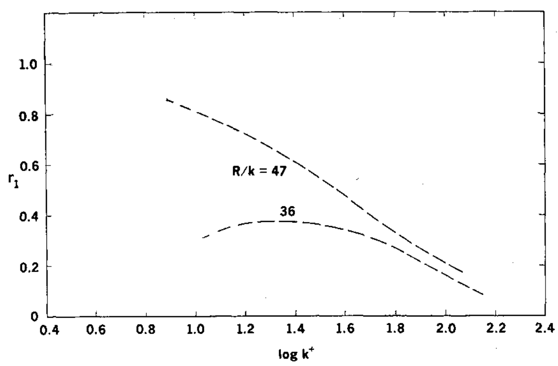

Fig. 4 The implied form of r_1 vs $\log k^+$ according to Spangler's faired data for R/k = 47 and 36.

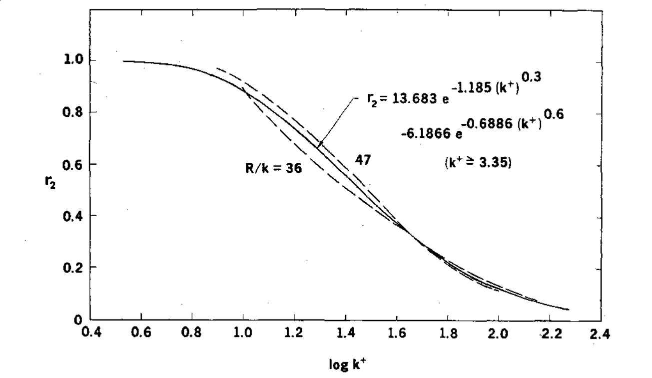

Fig. 5 The implied form of r_2 vs logk^+ according to Spangler's data for R/k = 47 and 36, together with a fitted exponential form.

Figure 3 reproduces Spangler's data[3] on the effects of uniform machined roughness on pipe flow of a 31 wppm solution of polyacrylamide P-295, as plotted by Poreh.[2] The assignment of the α value of 17.2, the v_{*crit} of 0.0968 fps, and the equivalent sand-grain roughness heights are explained in Ref. 3. The dashed curves through the data for $R/k = 36$ and 47 are our fairings (R is pipe radius). We consider the data for $R/k = 18$ to allow the interpretation that the behavior in that case is Newtonian.

The implied forms of r_1 and r_2 vs k^+ have been inferred from the data in Fig. 3 using Eq. (6) with (7) or (8), for the values of kv_{*crit}/ν determined from Fig. 7b of Ref. 3. The results are shown in Figs. 4 and 5; the dashed curves correspond to the faired curves in Fig. 3.

The results for the $\Delta B_m'$ model of Eq. (8) are much more promising than those for the model of Eq. (7). An exponential form has been fitted to the fairly coincident data curves, as shown in Fig. 5, assuming that $r_2 = 1.0$ for $k^+ \leq 3.35$.

The resultant behavior of ΔB according to Eqs. (6) and (8) for the case of $\alpha = 17.2$ is shown in Fig. 6, which should be compared with Fig. 2. The new model displays a much smoother onset of roughness effect, appreciably less roughness degradation of ΔB for small values of kv_{*crit}/ν, and the Newtonian-fluid value of k^+ at roughness effect onset, independent of α and v_{*crit}. It also displays a polymer effect cutoff at high kv_{*crit}/ν, due to r_2 keeping $r_2 v_*/v_{*crit} < 1$ for all k^+, which is consistent with our interpretation of the data for $R/k = 18$ in Fig. 3. Of course, we must emphasize that only further work can determine if the model is really useful; if r_2 depends on polymer type, concentration, and type of roughness; and how the onset of roughness effect depends on k^+.

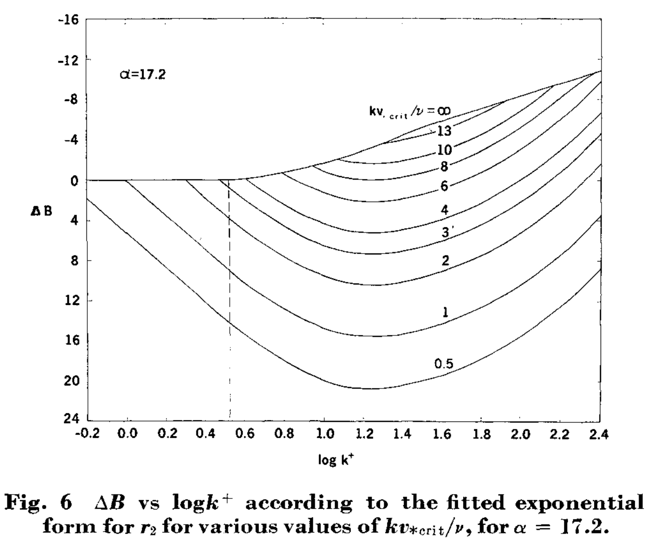

Fig. 6 ΔB vs logk^+ according to the fitted exponential form for r_2 for various values of kv_{*crit}/ν, for $\alpha = 17.2$.

Conclusions

Poreh's proposed model for the log law ΔB shift due to polymeric friction reduction with wall roughness contains an anomaly for low values of kv_{*crit}/ν. The peculiarly abrupt onset of roughness effect arises from an otherwise insignificant detail in his model for the Newtonian-fluid case. Some other possible models which display a smooth onset of roughness effect are considered. One of the two models tested with and fitted to Spangler's data seems to merit further investigation. It implies much less degradation of ΔB for small values of kv_{*crit}/ν than Poreh's model.

References

[1] Poreh, M., "Flow of Dilute Polymer Solutions in Rough Pipes," *Journal of Hydronautics*, Vol. 4, No. 4, Oct. 1970, pp. 151–155.

[2] Meyer, W. A., "A Correlation of the Frictional Characteristics for Turbulent Flow of Dilute Non-Newtonian Fluids in Pipes," *American Institute Chemical Engineers Journal*, Vol. 12, No. 3, May 1966, pp. 522–5.

[3] Spangler, J. G., "Studies of Viscous Drag Reduction with Polymer Including Turbulent Measurements and Roughness Effects," *Viscous Drag Reduction*, edited by C. S. Wells, Plenum Press, New York, 1969, p. 131.

Reply by Author to A. G. Fabula and D. M. Nelson

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The technical comment by Fabula and Nelson on the "Flow of Dilute Polymer Solutions in Rough Pipes"[1] discusses some limitations of the model proposed in Ref. 1 and presents an attempt to describe the available data by different mathematical expressions.

I fully agree with the comment that the function $p(k/\delta)$ chosen to approximate analytically the average data of Nikuradse changes abruptly in the neighborhood of k/δ =

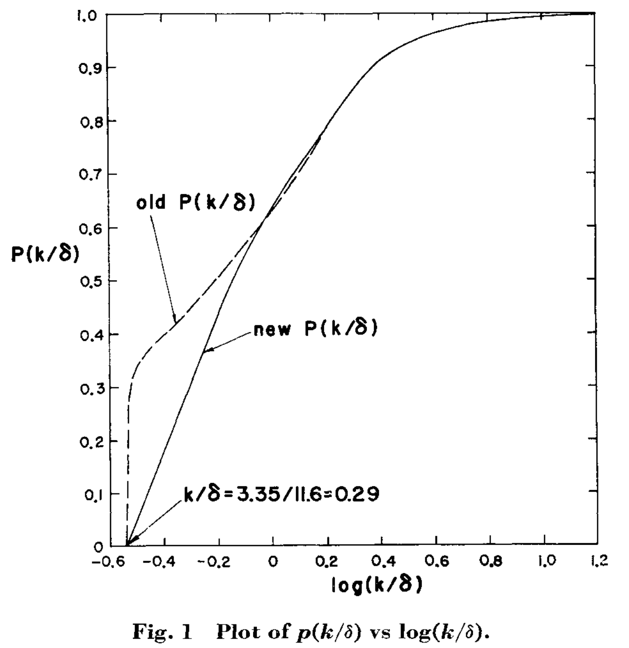

Fig. 1 Plot of $p(k/\delta)$ vs log(k/δ).

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